SWOT INSTITUTE
CONTINUITY AND DIFFERENTIABILITY
XII-TEST

Time : 1 hr.

Find the values of k so that the function f is continuous at the indicated point in Question No. 1 and 2.

1. $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{if } x \neq \dfrac{\pi}{2} \\ 3, & \text{if } x = \dfrac{\pi}{2} \end{cases}$ at $x = \dfrac{\pi}{2}$

2. $f(x) = \begin{cases} kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$ at x = 5.

3. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases}$$ is a continuous function.

4. Determine if f defined by

$$f(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$ is a continuous function ?

5. For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 3 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$

continuous at x = 0 ? What about continuity at x = 1 ?

6. Discuss the continuity of the function f where f is defined by

$$f(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$$

7. If x and y are connected parametrically by the equation given in Question, without eliminating the parameter, Find $\dfrac{dy}{dx}$.

$$x = a\left(\cos t + \log\tan\frac{t}{2}\right)\ y = a\sin t$$

8. If $y = \sin^{-1}x$, show that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = 0$.

9. If y = 3 cos (log x) + 4 sin (log x), show that $x^2 y_2 + xy_1 + y = 0$

10. If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$.

11. Differentiate the following w.r.t. x

(a) $\cos^{-1}(\sin x)$ (b) $\tan^{-1}\left(\dfrac{\sin x}{1 + \cos x}\right)$ (c) $\sin^{-1}\left(\dfrac{2^{x+1}}{1 + 4^x}\right)$

12. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

13. If cos y = x cos (a + y), with cos a $\neq \pm 1$, prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{\sin a}$.

14. If $y = e^{a\cos^{-1}x}$, $-1 \leq x \leq 1$, show that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$