

# SWOT INSTITUTE

## RELATIONS AND FUNCTIONS

### XII-TEST

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Time : 1 hr.

1. Show that the relation  $R$  in  $\mathbf{R}$  defined as  $R = \{(a, b) : a \leq b^3\}$ , is neither reflexive nor transitive and nor symmetric.
2. Let  $R$  be the relation in the set  $\{1, 2, 3, 4\}$  given by  $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$ . Choose the correct answer.  
(A)  $R$  is reflexive and symmetric but not transitive.  
(B)  $R$  is reflexive and transitive but not symmetric.  
(C)  $R$  is symmetric and transitive but not reflexive.  
(D)  $R$  is an equivalence relation.
3. Let  $S = \{1, 2, 3\}$ . Determine whether the function  $f : S \rightarrow S$  defined as below have inverse, Find  $f^{-1}$ , if it exists,  
(a)  $f = \{(1, 1), (2, 2), (3, 3)\}$   
(b)  $f = \{(1, 1), (2, 1), (3, 1)\}$   
(c)  $f = \{(1, 3), (3, 2), (2, 1)\}$ .
4. Show that  $f : [-1, 1] \rightarrow \mathbf{R}$ , given by  $f(x) = \frac{x}{(x+2)}$  is one-one. Find the inverse of the function  $f : [-1, 1] \rightarrow \text{Range } f$ .  
(Hint : For  $y \in \text{Range } f$ ,  $y = f(x) = \frac{x}{(x+2)}$ , for some  $x$  in  $[-1, 1]$ , i.e.,  $x = \frac{2y}{(1-y)}$ )
5. Consider  $f : \mathbf{R}_+ \rightarrow [-5, \infty)$  given by  $f(x) = 9x^2 + 6x - 5$ . Show that  $f$  is invertible with  $f^{-1}(y) = \left( \frac{(\sqrt{y+6})-1}{3} \right)$ .
6. For each operation  $*$  defined below, determine whether  $*$  is binary, commutative or associative.  
On  $\mathbf{Z}^+$ , define  $a * b = 2^{ab}$ .
7. Let  $A = \mathbf{N} \times \mathbf{N}$  and  $*$  be the binary operation on  $A$  defined by  
 $(a, b) * (c, d) = (a + c, b + d)$
8. Determine which of the following binary operations on the set  $\mathbf{R}$  are associative and which are commutative :  
 $a * b = \frac{(a+b)}{2} \quad \forall a, b \in \mathbf{R}$ .